Up and Down

Vertical Dilations of Quadratic Functions

2.3

LEARNING GOALS

In this lesson, you will:

- Graph quadratic functions through vertical dilations.
- Identify the effect on a graph by replacing *f*(*x*) by *Af*(*x*).
- Write quadratic functions given a graph.

KEY TERMS

- vertical dilation
- vertical stretching
- vertical compression
- reflection
- line of reflection

Have you ever heard that people shrink as they get older? Shrinking can occur from a number of reasons like osteoporosis, or because of the spine compressing over time. What you might not know is that everybody shrinks, and everybody also stretches every single day! This stretching and shrinking occurs because of little discs in a person's spine. They are filled with water, acting as the body's shock absorbers. As the day passes, these little discs lose water, compressing the spine. Then during sleep, the water is replenished, and the spine stretches back to its original size.

Although elderly people do "shrink," and although people expand and contract on a daily basis, people have actually been getting taller. One proof of this observation is the height of doorways in 18th century homes—they were not very tall! This is because people were on average, shorter than today. Over the last 150 years, the average height of a person has increased by 4 inches.

Does this mean that 300 years from now people will be an average of 8 inches taller than what most people are now? Do you think that the average height will eventually reach 9 or 10 feet? Are we destined to become giants, or do you think that this trend will stop?

PROBLEM 1 Vertical Stretching and Compressing

Now, let's explore the effects of the *A*-value in the transformational function.

$$g(x) = Af(B(x - C)) + D$$

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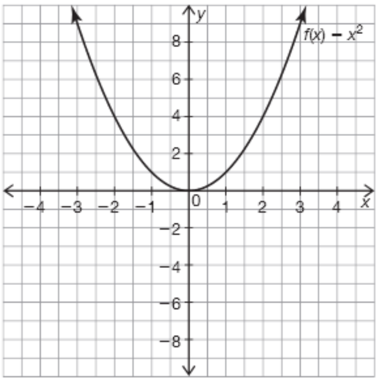
1. Analyze the transformational function. Where is the *A*-value positioned in terms of the function *f*: inside or outside the function? Based on the position of the *A*-value, do you think it will affect the *x*-values or the *y*-values? Explain your reasoning.

Let's consider various *A*-values to understand the effects on the basic function $f(x) = x^2$.

2. Use a graphing calculator to graph each quadratic function with $A > 0$. Sketch and label the graphs.

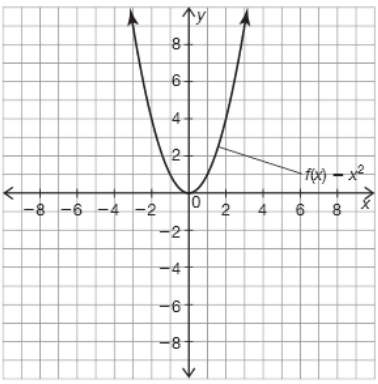
a.

$A \geq 1$
$2x^2$
$3x^2$
$4x^2$



b.

$0 < A < 1$
$\frac{1}{4}x^2$
$\frac{1}{2}x^2$
$\frac{3}{4}x^2$



c. Consider the different A-values. How do the graphs in part (a) compare to those in part (b)?

d. Does the A-value affect the x-values or y-values? Explain your reasoning.

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A **vertical dilation** is a type of transformation that stretches or compresses an entire figure or graph. In a vertical dilation, notice that the y-coordinate of every point on the graph of a function is multiplied by a common factor, A. You can also think about this as either a vertical stretch or vertical compression. **Vertical stretching** is the stretching of the graph away from the x-axis. **Vertical compression** is the squeezing of a graph towards the x-axis.

3. Is a dilation the type of transformation that preserves both the size and shape of a function? Explain your reasoning.

4. Consider the function $f(x) = \frac{3}{2}x^2$.
Dan says that the graph of this function will look more compressed than the graph of $f(x) = x^2$ because the A-value is a fraction. Jeannie says that the graph of this function will look more stretched because the A-value is greater than 1.
Who is correct? Explain your reasoning.

5. Choose a term that identifies the effect on the graph of replacing $f(x)$ with $Af(x)$:

vertical stretch	vertical compression

a. $A \geq 1$ __________________

b. $0 < A < 1$ __________________

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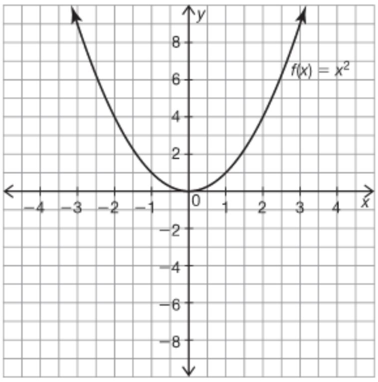
6. Given the basic function $f(x) = x^2$. Use a graphing calculator to graph each quadratic function with $A < 0$. Sketch and label the graphs. Also, use the TABLE feature and analyze each table of values.

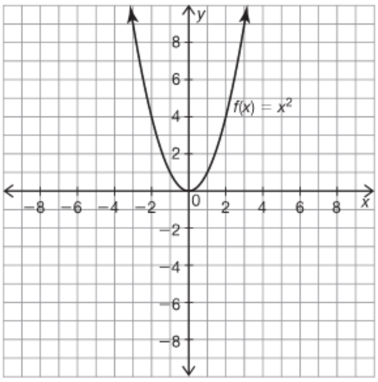
a.

$A \leq -1$
$-2x^2$
$-3x^2$
$-4x^2$



b.

$-1 < A < 0$
$-\frac{1}{4}x^2$
$-\frac{1}{2}x^2$
$-\frac{3}{4}x^2$



c. How did $A < 0$ affect the graph of the basic function.

d. How do the graphs in part (a) compare to the graphs in Question 2, part (a)?

e. How do the graphs in part (b) compare to the graphs in Question 2, part (b)?

A **reflection** of a graph is a mirror image of a graph across its *line of reflection*. A **line of reflection** is the line that a graph is reflected across.

7. Identify the line of reflection for each graph in Question 6.

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Compared with the graph of $y = f(x)$, the graph of $y = Af(x)$ is:

- vertically stretched by a factor of $|A|$ if $|A| > 1$.
- vertically compressed by a factor of $|A|$ if $0 < |A| < 1$.

If $A < 0$, then the graph is also reflected across the x-axis.

8. Given $y = f(x)$, write the coordinate notation represented in $y = Af(x - C) + D$.

$(x, y) \rightarrow$ ______________________________

2

9. Use a graphing calculator to sketch each set of equations on the same coordinate plane.

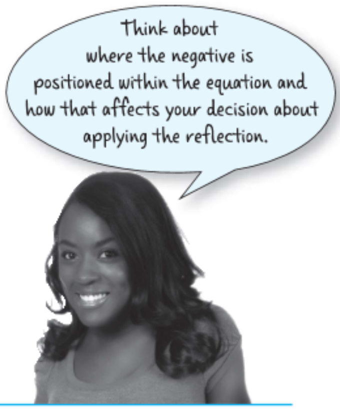
a. $y_1 = x^2 + 2$
$y_2 = -x^2 + 2$

How do the graphs of y_1 and y_2 compare?

b. $y_1 = x^2 + 2$
$y_3 = -y_1$

How do the graphs of y_1 and y_3 compare?

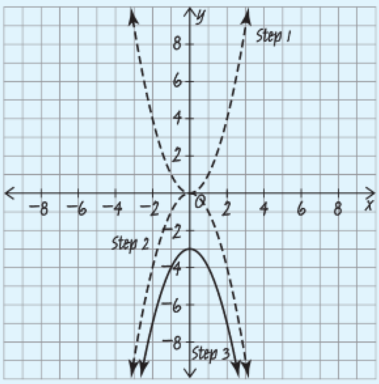
10. Explain the differences between the lines of reflections used to produce y_2 and y_3 in Question 8.



11. Christian, Julia, and Emily each sketched a graph of the equation $y = -x^2 - 3$ using different strategies. Provide the step-by-step reasoning used by each student.

Christian

$A = -1$ and $D = -3$



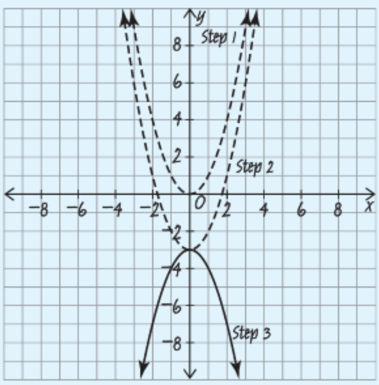
Step 1:

Step 2:

Step 3:

Julia

$D = -3$ and $A = -1$



Step 1:

Step 2:

Step 3:

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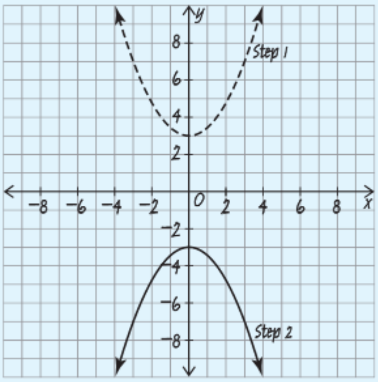
Emily

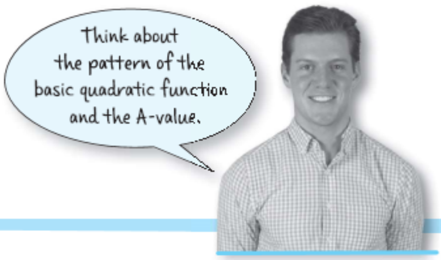
I rewrote the equation as $y = -(x^2 + 3)$.



Step 1:

Step 2:

Given $y = f(x)$ is the basic quadratic function, you can use reference points to graph $y = Af(x - C) + D$ without the use of technology.



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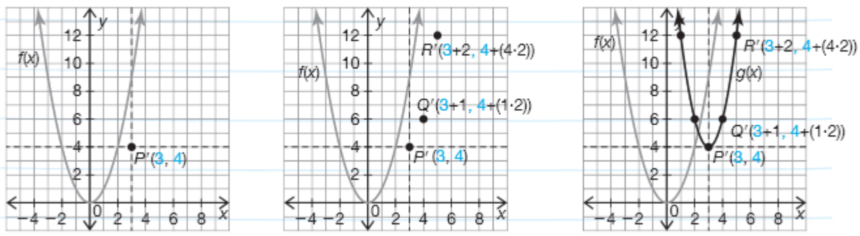
Given $f(x) = x^2$

Graph the function $g(x) = 2f(x - 3) + 4$

You can use reference points for $f(x)$ and your knowledge about transformations to graph the function $g(x)$.

From $g(x)$, you know that $A = 2$, $C = 3$, and $D = 4$.

The vertex for $g(x)$ will be at (3, 4). Notice $A > 0$, so the graph of the function will vertically stretch by a factor of 2.



First, plot the new vertex, (*C*, *D*). This point establishes the new set of axes.

Next, think about the reference points for the basic quadratic function and that $A = 2$.

To plot point Q' move right 1 unit and up, not 1, but 1×2 units from the vertex P' because all y-coordinates are being multiplied by a factor of 2. To plot point R' move right 2 units from the P' and up, not 4, but 4×2 units.

Finally, use symmetry to complete the graph.

2

12. Analyze the worked example.

a. Use coordinate notation to represent how the *A*-, *C*-, and *D*-values transform the basic quadratic function to generate $g(x) = 2f(x - 3) + 4$.

$(x, y) \rightarrow$ ____________________________

b. Use the coordinate notation from part (a) to complete the table of values to verify the graph.

Reference Points of Basic Quadratic Function	→	Corresponding Points on $g(x)$
(0, 0)	→	
(1, 1)	→	
(2, 4)	→	

c. Rewrite $g(x)$ in terms of x.

13. Suppose function $d(x)$ has the same C- and D-values the function $g(x)$ in the worked example, but its A-value is $\frac{1}{2}$.

a. Write $d(x)$ in terms of $f(x)$.

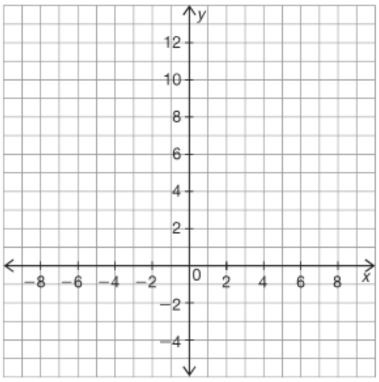
b. How would the graph of $d(x)$ compare to the graph of $g(x)$? Explain your reasoning.

2

c. Complete the table of values.

Reference Points of Basic Quadratic Function	→	Corresponding Points on $d(x)$
(x, y)	→	
(0, 0)	→	
(1, 1)	→	
(2, 4)	→	

d. Sketch the graph of $d(x)$.



e. Rewrite $d(x)$ in terms of x.

14. Suppose a function, $r(x)$, has the same C- and D-values the function $g(x)$ in the worked example, but its A-value is -2.

a. Write $r(x)$ in terms of $f(x)$.

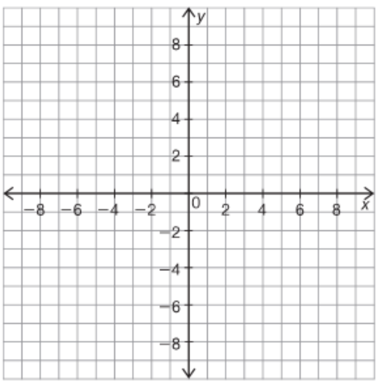
b. How would the graph of $r(x)$ compare to the graph of $g(x)$? Explain your reasoning.

2

c. Complete the table of values.

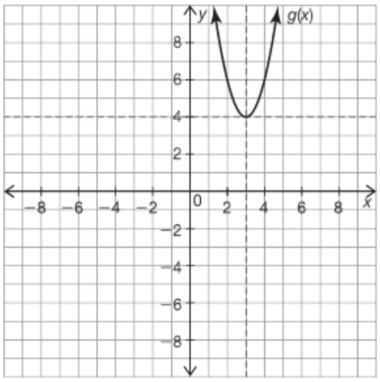
Reference Points of Basic Quadratic Function	→	Corresponding Points on $r(x)$
(x, y)	→	
(0, 0)	→	
(1, 1)	→	
(2, 4)	→	

d. Sketch the graph of $r(x)$.



e. Rewrite $r(x)$ in terms of x.

15. Given the graph of $g(x) = 2(x - 3)^2 + 4$ from the worked example.

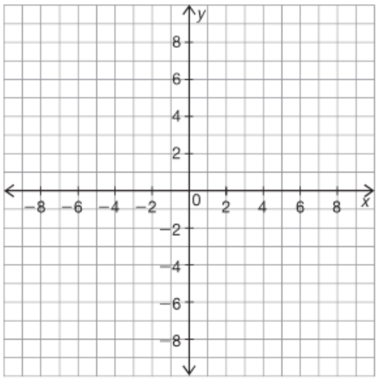


a. Graph $m(x) = -g(x)$.

b. How is the graph of $m(x)$ the same or different from the graph of $r(x)$ in Question 13?

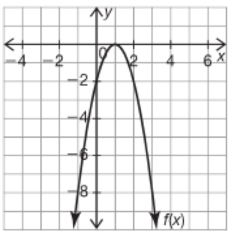
c. Rewrite $m(x)$ in terms of x.

16. Graph $f(x) = \frac{1}{2}(x - 1)^2 + 3$.

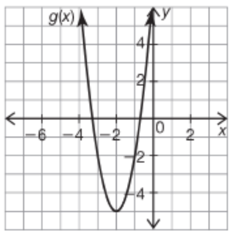


17. Write the functions that represent each graph.

a.



b.



$f(x) =$ ______________

$g(x) =$ ______________

Be prepared to share your solutions and methods.